AERODYNAMICS OF MOVING BELTS, TAPES, AND WEBS

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ABSTRACT

The mass of the air surrounding a moving belt leads to a number of forces normal to the belt: hydrodynamically induced inertia, Coriolis force, and centrifugal force. These forces act in concert with those generated by the belt's own inertia to govern out-of-plane dynamics. Their effect is to reduce stability and limit the safe running speed. These detrimental consequences can be reduced through management of the air flow around the belt. An analysis of the governing partial differential equations is presented; experiments on the aerodynamic interaction terms are reported; and a numerical simulation method for belt dynamics is outlined.

NOMENCLATURE

A, B, b, C General coefficients
c Phase velocity
d Width of belt
EI Bending stiffness of belt
F, f Forcing functions
f_n nth mode natural frequency
E_x Excitation frequency
L Length of belt span
M_{web} Mass per unit length of belt
M_a Added mass due to \frac{d^2 z}{dx^2} at constant \xi
M_b Added mass due to \frac{d^2 z}{dx^2} at constant \xi
M_c Added mass due to \frac{d^2 z}{dx^2} at constant \xi
M_d Added mass due to \frac{d^2 z}{dx^2} at constant \xi
M_e Added mass due to \frac{d^2 z}{dx^2} at constant x
M_f Added mass due to \frac{d^2 z}{dx^2} at constant x
M_g Added mass due to \frac{d^2 z}{dx^2} at constant x
T Tension
\tau Time
U Air speed
U_{div} Critical air speed for divergence
U_{fl} Critical air speed for flutter
V Running speed of belt
x Coordinate in the direction of belt motion
y Coordinate across the belt width
z Out-of-plane deflection of the belt
\delta' Displacement thickness of boundary layer
= \frac{1}{V}\int_{z}^{z_0} U(z) \, dz
\theta Momentum thickness of boundary layer
= \frac{1}{V}\int_{z}^{z_0} U'(z) \, dz
\lambda Wavelength
\xi x-direction coordinate moving with belt
\rho Air density
\omega Angular frequency

INTRODUCTION

There is a large class of moving belts and tapes -- ranging from power transmission systems to webs of paper or plastic running through dryer ovens -- which are subject to flutter, billowing, or other out-of-plane motions. These motions frequently have detrimental effects, and place limits on the speeds at which power or production machinery can be operated. It is important for the design engineer to understand the physics involved in order to anticipate and avoid vibration, flutter, and instabilities. One of the dynamic effects is the interaction of the solid material with the surrounding fluid.

There are three circumstances under which the mass of the surrounding gas or liquid becomes important to the moving sheet: if the translating material is wide, or if it is thin, or if the surrounding fluid is dense. In these cases the hydrodynamic inertia of the air or liquid becomes large compared to the mass of the belt itself, even when the fluid is practically inviscid.

We have taken a three-part approach to the problem of predicting the influence of aerodynamic interactions: first, we present an analysis which describes the physical process through a differential equation in the form of a streamline equation with added hydrodynamic inertia terms. In this way, the pertinent coefficients are identified, the form of the solution is obtained, and the critical velocity of instability quantified. Secondly, we report experiments
for stationary sheets in a wind tunnel, in order to verify the theory, the magnitude of the hydrodynamic inertia coefficients, and the nature of the instabilities. Finally, we develop numerical procedures for implementing the theory and using the inertia coefficients in the simulation of more complex situations.

**ANALYSIS -- MOVING COORDINATES**

For an observer moving with the belt, the governing equation for a belt surrounded by a fluid is

\[
(M_{\text{web}} + M_s) \frac{\partial^2 \xi}{\partial t^2} + 2M_b \frac{\partial \xi}{\partial t} + (M_s + M) \frac{\partial^2 \xi}{\partial t^2} \times V + E \frac{\partial^2 \xi}{\partial \xi^2} = f(\xi, t) \tag{Eqn. 1}
\]

This is essentially the same equation as derived by Lighthill (1960) for a swimming fish. In the first term, the differentiation is taken at a constant \(\xi\)-location on the belt. Acceleration \(\frac{\partial^2 \xi}{\partial t^2}\) is redefined not only by the mass of the belt \(M_{\text{web}}\), but also by the mass of the air \(M_b\) impelled by the belt when it is displaced normal to the plane of the belt, in the \(z\)-direction. If the air moves out of the way mostly around the sides of the belt in a plane normal to the axis of the belt, and not along the axis or around the ends of the belt (Lighthill's "slender fish" assumption), the added mass may be obtained by integrating the kinetic energy in the two-dimensional flow field caused by a reference out-of-plane velocity of belt, and setting the result equal to one-half times \(M_b\) times the square of the reference velocity (Chang and Morelli, 1991). For potential flow in a large space, this hydrodynamic inertia was shown by Lamb (1945) to be

\[
M_s = \frac{\pi \rho d^4}{4} \tag{Eqn. 2}
\]

where \(\rho\) is the density of the surrounding fluid and \(d\) is the width of the belt. Clearly, the importance of this mass varies with the application; for a roller chain in air, \(M_s\) is much smaller than \(M_{\text{web}}\); for a paper web 30 feet wide, \(M_s\) is much larger than \(M_{\text{web}}\).

The second term in Eqn. 1, with the mixed derivative, is the Coriolis term. If the \(\xi\)-coordinate is taken opposite to the motion of the belt (back from the nose of Lighthill’s swimming fish) and therefore in the direction of the relative velocity of the stagnant fluid, the plus sign applies; if \(\xi\) is taken in the direction of \(V\), the absolute velocity of the belt itself, the negative sign applies. The magnitude of \(M_b\) was shown by Lighthill to be equal to \(M_b\) in Eqs. 2 if the surrounding fluid is stationary. If some of the fluid next to the belt is dragged along with it, \(M_b\) will be somewhat less; indeed, if all the fluid were moving along with the belt, \(M_b\) would be equal to zero.

The third term in Eqn. 1 contains the centrifugal force term. Again, \(M_s\) is equal to \(M_s\) in Eqn. 2 if the surrounding fluid is stagnant; if a boundary layer of fluid is dragged along with the belt, \(M_s\) will be slightly less; if all the fluid were moving with the belt, \(M_s\) would be zero. The centrifugal force tends to promote excursions in \(z\), opposing the restoring force due to the tension \(T\). If the belt has significant stiffness, there will also be a restoring force due to the bending stiffness \(E\).

Some limiting cases of this equation are as follows:

1. **Case 1:** If the air mass is negligible, we are left with the familiar string or beam equation

\[
M_s \frac{\partial^2 \xi}{\partial t^2} - T \frac{\partial^2 \xi}{\partial t^2} + E \frac{\partial^2 \xi}{\partial \xi^2} = f(\xi, t) \tag{Eqn. 3}
\]

2. **Case 2:** If all the surrounding air moves with the belt, we are left with the air-loaded string or beam equation

\[
(M_{\text{web}} + M_b) \frac{\partial^2 \xi}{\partial t^2} - T \frac{\partial^2 \xi}{\partial t^2} + E \frac{\partial^2 \xi}{\partial \xi^2} = f(\xi, t) \tag{Eqn. 4}
\]

3. **Case 3:** If the surrounding space is large, the belt or swimming fish is slender, and the fluid is stationary,

\[
(M_{\text{web}} + M_b) \frac{\partial^2 \xi}{\partial t^2} + 2M_b \frac{\partial \xi}{\partial t} + (M_s + M) \frac{\partial^2 \xi}{\partial t^2} \times V + E \frac{\partial^2 \xi}{\partial \xi^2} = f(\xi, t) \tag{Eqn. 5}
\]

The usefulness of the moving coordinate system is to look at the propagation of local disturbances, as long as they are away from the supports or rollers. If the stiffness term is negligible, several initial-condition solutions are already known. In Case 1, waves traveling in either direction at a relative velocity equal to \(\frac{T}{M_s + M}\) (Fig. 1a) automatically satisfy the differential equation. This is also true in Case 2, but the propagation velocity is reduced to \(\frac{T}{M_{\text{web}} + M_s}\).

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![Fig. 1a Propagation of a disturbance along a travelling string in the absence of surrounding fluid](image-url)
In Case 3, wave shapes are not preserved; the propagation velocities (relative to the belt) will be greater in the direction counter to belt motion than in the direction of belt motion (Fig. 1b); and the restoring force will fall if \( T \) is less than \( M_4 V^2 \). Note that \( T \) is the actual tension in the belt; the apparent tension obtained from reaction forces on rollers is approximately \( (T - M_4 V^2) \).

![Fig. 1b](image)

**Fig. 1b** Propagation of a disturbance along a traveling string in the presence of surrounding fluid

For Case 4, Eqsns. 10 and 11 in the text, \( v_4 = v_0 \)

**ANALYSIS -- STATIONARY COORDINATES**

In order to incorporate support conditions at the ends of a span, it is useful to write the differential equation in fixed coordinates, obtainable from Eqs. 1 by coordinate transformation:

\[
(M_{web} + M_4) \frac{\partial^2 V}{\partial t^2} = 2(M_{web} + M_4) \frac{\partial^2 V}{\partial x \partial t} + \left[(M_{web} + M_4) V^2 - T \frac{\partial^2 Z}{\partial x \partial t} + EI \frac{\partial^2 Z}{\partial x^2} = F(x, t) \right] \quad \text{(Eqn. 6)}
\]

This is essentially the same equation as derived by Housner (1952) for fluid flowing inside a pipe. In the first term, the differentiation is now taken at constant \( x \), as seen by a stationary observer, so that the value of the derivative is different than in Eqn. 1. However, the mass coefficients are the same and we find that \( M_4 \) has the same value as \( M_4 \) in Eqn. 2 if the waves in the response mode are much longer than the width of the belt \( d \) (the slenderness assumption). On the other hand, if the response consists of short sinusoidal ripples with a length \( \lambda \) much shorter than the width \( d \) (so that the air moves mostly in the \( x \)-plane, rather than around the sides of the belt), the added mass is \( \rho a/\pi \). Thus the added mass depends on the shape of the response mode.

The second term, with the mixed derivative, is the Coriolis term. The positive sign is for the belt velocity in the positive \( x \)-direction. \( M_4 \) is zero if the surrounding air is stationary; has the moderate value \( 2p \delta a \) if thin boundary layers (much smaller than \( d \)) of displacement thickness \( \delta \) (White, 1974) are dragged along with the belt; and has the same value as \( M_4 \) if all the surrounding fluid moves with velocity \( V \). \( M_4 \) can become negative if the surrounding air is forced into flow counter to the belt velocity \( V \).

The third term contains centrifugal forces due both to \( M_{web} \) and \( M_4 \). \( M_4 \) is zero if the surrounding air is stationary; has the small value \( 2p \delta a \) if thin boundary layers of momentum thickness \( \theta \) (White, 1974) are dragged along with the belt; and has the same value as \( M_4 \) if all the surrounding fluid moves along with velocity \( V \). Comparing Eqns. 1 and 6, we note that \( M_1 = M_2 = M_3 = M_3 - M_4 \); \( M_4 = M_4 - 2M_4 + M_4 \). For viscous flows (thick boundary layers), \( M_1, M_2, \) and \( M_3 \) are difficult to obtain, since potential-flow superposition is no longer applicable.

It should be noted that the boundary-layer thicknesses needed here cannot be predicted by the usual flat-plate formulation, in which the boundary layer grows toward infinity only as the plate's \( x \)-coordinate goes to infinity. The analogy is not correct since the moving belt does not have a leading edge. Therefore, the moving-belt boundary layer does not grow with distance from a leading edge, but with time, and grows toward infinity forever or until the momentum transfer is balanced by fluid friction on outer boundaries. The more nearly correct boundary-layer growth analogy from classical fluid physics, therefore, is the transient flow field around a rotating cylinder or sphere. Equilibrium flow fields require the consideration of outer boundaries to the flow. A simple example of a web translating through a drying oven has been calculated by Chang and Moretti (1991).

Some limiting cases for Eqn. 6 are as follows:

1. **Case 1:** If the air mass is negligible, we are left with the familiar threadline equation

\[
(M_{web} + M_4) \frac{\partial^2 V}{\partial t^2} + 2(M_{web} + M_4) \frac{\partial^2 V}{\partial x \partial t} + 04_{web} V^2 - T \frac{\partial^2 Z}{\partial x \partial t} + EI \frac{\partial^2 Z}{\partial x^2} = F(x, t) \quad \text{(Eqn. 7)}
\]

2. **Case 2:** If all the surrounding air moves with the belt, we are left with an added-mass threadline equation

\[
(M_{web} + M_4) \frac{\partial^2 V}{\partial t^2} + 2(M_{web} + M_4) \frac{\partial^2 V}{\partial x \partial t} + 04_{web} V^2 - T \frac{\partial^2 Z}{\partial x \partial t} + EI \frac{\partial^2 Z}{\partial x^2} = F(x, t) \quad \text{(Eqn. 8)}
\]

3. **Case 3:** If all the fluid is stationary etc. as above, i.e., if the mass of air in the boundary layers is negligible,

\[
(M_{web} + M_4) \frac{\partial^2 V}{\partial t^2} + 2(M_{web} + M_4) \frac{\partial^2 V}{\partial x \partial t} + 04_{web} V^2 - T \frac{\partial^2 Z}{\partial x \partial t} + EI \frac{\partial^2 Z}{\partial x^2} = F(x, t) \quad \text{(Eqn. 9)}
\]
Case 4: A special symmetry can be seen for Case 3 if $M_{\text{web}} = M_1$; Eqns. 1 or 5 in $\xi$-coordinates then becomes

$$2M \frac{\partial^2 \xi}{\partial t^2} + 2MV \frac{\partial^2 \xi}{\partial \eta \partial t} + (MV^2 - T) \frac{\partial^2 \xi}{\partial \eta^2} + EI \frac{\partial^4 \xi}{\partial \eta^4} = F(\xi, t)$$

(Eqn. 10)

and Eqns. 6 or 9 in $x$-coordinates becomes

$$2M \frac{\partial^2 \xi}{\partial t^2} + 2MV \frac{\partial^2 \xi}{\partial x \partial t} + (MV^2 - T) \frac{\partial^2 \xi}{\partial x^2} + EI \frac{\partial^4 \xi}{\partial x^4} = F(x, t)$$

(Eqn. 11)

Since the coefficients are identical in both coordinate systems, for this special case, we see that the forward propagation of waves relative to the belt must be the same as the backward propagation of waves in absolute coordinates; and the backward propagation on waves relative to the belt the same as the forward propagation in absolute coordinates, as sketched in Fig. 1b. Thus we see how hydrodynamic loading of the belt will retard forward propagation of disturbances.

**ANALYTICAL SOLUTIONS**

For Case 1 (Eqn. 7) and also Case 2 (Eqn. 8) with negligible EI, the $n^{th}$ mode complementary solution for a span of length $L$ is (Chang and Moretti, 1991):

$$z = \cos(2\pi f_n t) \left[ \hat{n} \frac{V}{c} \right] \sin \left( \frac{\hat{n} \pi x}{L} \right)$$

(Eqn. 12)

where $c = \sqrt{TM_{\text{web}}}$ for Case 1 and $\sqrt{TM_{\text{web}} + M_1}$ for Case 2, and

$$f_n = \frac{\hat{n} \pi}{2L} \left[ 1 - \left( \frac{V}{c} \right)^2 \right]$$

(Eqn. 13)

The shape of the first-mode motion is shown by Fig. 2; there is a phase difference between the motion at different stations along the belt.

The dependence of natural frequency on web speed is plotted in Fig. 3: when it goes to zero (at $z = V$), stability is lost and the belt will diverge or "sail" into a "billowing" state. Adding aerodynamic terms as in Eqn. 6, with the EI-term neglected, we find that the effect of each added-cross term is as indicated by the arrows in Fig. 3: $M_1$ acts to reduce natural frequency; $M_2$ reduces the critical "buckling" velocity where the natural frequency goes to zero; $M_3$ deflects the curve between the intercepts in either direction, depending on flow direction. Clearly the management of $M_3$ (and to a lesser extent $M_2$) by management of the flow field around the belt is important to maintaining a margin of belt stability at high running speeds.

**Fig. 2** Free vibration of a traveling threadline

**Fig. 3** Effect of aerodynamic terms on the dynamics of a web
EXPERIMENTS

To verify the effects of air flow, we performed experiments on stationary webs in a wind tunnel, as described by Chang (1990). For flexible webs or long wavelengths,

$$|E| \frac{d^2 y}{dx^2} | < \left| T \frac{d^2 z}{dx^2} \right|,$$

and the differential equation reduces to

$$\left( M_{\text{web}} + M_j \right) \frac{d^2 z}{dx^2} + 2 M_j U \frac{d^2 z}{dx^2} + \left( M_j v^2 - T \right) \frac{d^2 y}{dx^2} = F(x, t) \quad \text{(Eqn. 14)}$$

When the restoring force term vanishes, the web experiences static instability. The main driving force for static instability is the centrifugal force. A slender web in a uniform flow field will diverge at

$$U_{\text{div}} = \frac{4 T}{\sqrt{\pi p d}} \qquad \text{(Eqn. 15)}$$

Assuming harmonic motion of the web, one can derive the critical flutter speed for a slender web as

$$U_{\text{fl}} = \sqrt{\frac{T}{p d}} \sqrt{1 + \frac{4 M_{\text{web}}}{M_j}} \quad \text{(Eqn. 16)}$$

For the tests, we used paper webs ($M_{\text{web}} = 1.08 \times 10^{-4}$ lb/ft$^2$, $E = 9.9 \times 10^3$ psi) in four different sizes: 3" x 10", 3" x 20", 5" x 10", and 6" x 20". The test section of the wind tunnel is 24.5 inches wide and 16.25 inches high. The tested paper webs were narrow enough compared to the tunnel dimensions that the webs could be considered to be in an infinite, uniform flow field. The leading and trailing edges were restrained from vertical motion; while the other two edges were free to move. In order to implement the desired boundary conditions of the web, a web holder shown in Fig. 4 was used. Tension was applied through the trailing edge.

As the air speed was increased from zero, the paper started to diverge at a critical flow speed; it bulged out either upward or downward. When the flow speed was increased further, the displacement was also increased; in some cases, a higher mode (multiple-wave) divergence occurred as shown in Fig. 5.

At flow speeds higher than the divergence speed, a low-speed traveling wave occurred. The oscillation frequency and amplitude grew with the flow speed into a pronounced flutter, which became violent at higher flow speeds. In some cases, static divergence was suddenly changed to flutter by a small increase of the flow speed. In other cases, there was a stable range between the static and dynamic instability regions.

Critical flow speeds for static instability and flutter were determined for each of test conditions, i.e., for given web dimensions and tension. Fig. 6 shows the stability boundaries of static divergence; each data point indicates a critical value of dynamic pressure above which bulging occurred. The compared theoretical curve is

$$q = \left( \frac{2}{k} \right) \left( \frac{U}{U_{\text{cr}}} \right)$$

which is a variation of Eqn. 15. The experimental results are higher than the expected values, but they show the same tendency.
The experimental results for flutter are compared with a theoretical curve in Fig. 7, where the curve is

$$\frac{q}{2} = \frac{pd}{2M_{web}V_d^2} \left( \frac{T}{d} \right)$$  \hspace{1cm} \text{(Eqn. 18)}

which is from Eqns. 16. The experimental values fall below the prediction curve.

![Fig. 7 Dynamic stability criteria of a web in an airflow](image)

**NUMERICAL SIMULATION**

If we neglect the EI-term (the influence of EI is discussed by Chang, 1990) we see that all the partial differential equations cited above have the basic form

$$A\ddot{z} + B\dot{z} + Cz = F$$  \hspace{1cm} \text{(Eqn. 19)}

where the excitation may enter from initial conditions, from support boundary oscillation (roller eccentricity), from an acoustic forcing function F, or from parametric variation in tension (included in C). If we leave off the last possibility for now, we see that we can eliminate the coefficients A by normalizing the equation. Then we can also eliminate C by changing the scaling of time by a factor equal to $V/C\lambda$, so that the equation is reduced to

$$\ddot{z} + b\dot{z} + c^2z = f$$  \hspace{1cm} \text{(Eqn. 20)}

where the remaining system parameter b is the previous B divided by $V\lambda$, and the forcing function f is F/V. For example, applying this to Case 1 and Eqn. 7, or Case 2 and Eqn. 8, we have $b=2V/(2\zeta - V^2)$, a dimensionless constant. Therefore, if we can compute this equation for a wide range of b, we can simulate a wide variety of V and c values.

A finite-difference formulation was chosen for the numerical simulation. The simplest algorithmic scheme is shown in Fig. 8, which shows a cluster of points in the (x-t) domain. Each grid intersection point P has E (east) and W (west) neighbors in the spatial direction, and new-time (prime) and old-time (minus) neighbors in the time direction. The simplest finite-difference scheme approximates the second-order derivatives centered at P, but for stability reasons it has been found useful to take a more "upstream" representation of the cross-derivative term in which the $\ddot{z}$ term (the Coriolis term) is centered to the west of P when the web running velocity V is large and positive (see Fox, 1990, and Fox and Lillig, 1991).

![Fig. 8 Finite-Difference Cluster of Points](image)

There is an inherent stability problem which can be demonstrated on the simple non-translating vibrating-string equation. An initial disturbance at a single point in the middle of the string will split and propagate in both directions at the phase velocity c if the time interval is chosen to be 1/c times the longitudinal division. This is the physically correct result. If the time interval is chosen larger, any disturbance, even a round-off error, will cause the solution to "blow up" into growing alternating values; the solution scheme becomes unstable. Conversely, if the time interval is chosen smaller than the correct value, the propagating waves, even those from real initial conditions, will progressively flatten and "smear" out. Looking at long-wave solutions, this "numerical diffusion" will not show up as dramatically, but will cause an apparent damping and decay of the modal solutions. For a translating string, there is no longer a single phase velocity (see Fig. 1b), and no single "correct" time interval for the computation.

Optimal time and space intervals, and degree of upstream differencing of the Coriolis term, were investigated by Fox (1990). Attention was focused on the dynamic response of running webs subjected to the oscillations because of initial displacement from their equilibrium positions, or because of transients introduced from sinusoidal movement at their upstream roller contact point (roller irregularity). As a step in developing numerical methods for out-of-plane dynamics, the one-dimensional partial differential equation, Eqn. 20 was converted into a computational algorithm by means of finite-difference formulations. The entire web begins to oscillate, and we are especially interested in the vibration modes.

Explicit finite-difference schemes for modeling one-dimensional, transient web dynamics were designed, developed and tested. A sequence of numerical experiments were performed to ascertain the effects of various parameters on the stability and accuracy of the numerical results. The parameters studied were time-step size and spatial domains grid size. Evaluation of variation of parameter results was performed and criteria for accurate web dynamic numerical simulations stipulated. The use of upstream differencing of the Coriolis cross-derivative term in Eqn. 20 for better computational stability was examined. Numerical simulations using a combination of upstream and central differencing were performed and analyzed. As an example, Fig. 9a shows instabilities rapidly showing themselves when Eqn. 20 is simulated using the theoretical maximum step-size for t, even when full upstream differencing is used for the Coriolis term. The instabilities were removed, at the expense of damping, and a stable computation followed when a five-fold reduction in t step-size was made, as illustrated in Fig. 9b. In this figure $x = 0$ and $x = L$ correspond to the upstream and downstream roller locations.
Upstream differencing of the Coriolis term to produce stability leads to numerical damping in the computations. A combination of upstream and central differencing yields stable results at larger time-steps than that required using full central differencing, as well as yielding less damping than that produced using full upstream differencing.

Webs subjected to oscillation because of upstream roller eccentricity were simulated via a sinusoidal oscillation being imposed at this location. The other end of the web was given a free zero-gradient condition, and was therefore permitted to 'float' if the further upstream oscillations propagated downstream sufficiently well. The faster the web is moving axially, the greater is this downstream propagation. As an example, Fig. 10 demonstrates the web dynamics with b (see Eqn. 20) equal to 0, 1.15, and 4.13 respectively in parts a, b and c. The computations here have upstream roller oscillation frequency four times the natural frequency of the web. Results at other oscillation frequencies are also available (see Fox, 1990). The extent of out-of-plane movement is clearly seen to magnify as the parameter b increases, this being the case as web running velocity V increases. These computations were performed with full upstream differencing of the Coriolis term, and reduced time-step size as recommended in the earlier study of initially deflected web dynamics.

In summary, it has been found that refinement of the numerical grid leads to results which converge to analytical results for stable computations. For a particular web running speed and computational duration, there exists a time-step limit for stability. Longer computational time durations as well as higher web running speeds require reduced time-step sizes for stable numerical results.
CONCLUSIONS

The results of this paper can be applied on several different levels. Most basically, the natural frequency of a belt is an indicator of its margin of stability; by using the theory to estimate the several added-mass coefficients and the lowest natural frequency, it is possible to gauge the risk of flutter at increasing web speeds. If it appears necessary for accuracy, the experiments can be extended to obtain added-mass coefficients for a variety of boundary conditions and flow distributions.

In addition to natural frequency, it is possible to obtain the shape and amplitude of web response for various excitations -- direct forcing by fluctuating pressure, oscillating boundaries due to roller eccentricity, or parametric excitation from fluctuating tension -- by means of the numerical simulation. The computations need to be iterative, because the added-mass coefficient may need to be adjusted as the wavelength of the response emerges.

Future challenges are extension of the simulation to wide belts, which may have non-uniform tension and deflection across their width, and study of the effect of transverse flow on edge flutter.

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